§4.1—Antiderivatives & Indefinite Integration

Suppose we have a function F whose derivative is given as $F'(x) = f(x) = x^2$. From your experience with finding derivatives, you might say that $F(x) =$ WHAT???? How can you check your answer?????

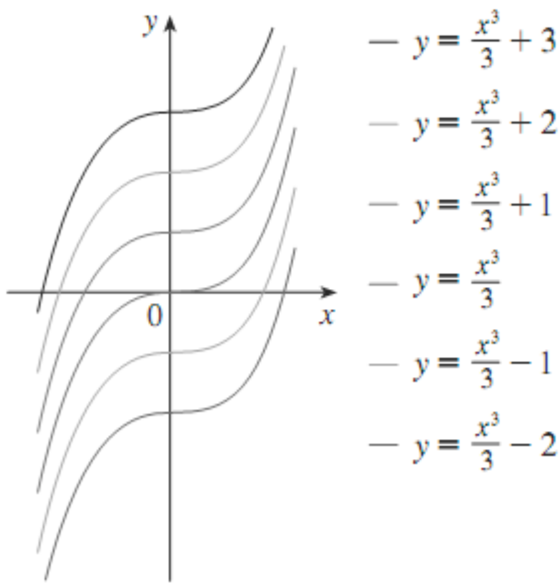
Congratulations, you have just found *an* **antiderivative**, F, of f.

Definition

A function F is *an* **antiderivative** of f on an interval I if $F'(x) = f(x)\ \forall x \in I$.

Notice that F is called AN antiderivative and not THE antiderivative. This is easily understood by looking at the example above.

Some antiderivatives of $f(x) = x^2$ are $F(x) = \frac{1}{3}x^3$, $F(x) = \frac{1}{3}x^3 + 3$, $F(x) = \frac{1}{3}x^3 - 2$, and $F(x) = \frac{1}{3}x^3 + \pi$ because in each case, $\frac{d}{dx}\left[F(x)\right] = x^2$.



Because of this we can say that the **<u>general antiderivative</u>** of a function $f(x)$ is

$$F(x) + C, \text{ where } C \text{ is an arbitrary constant.}$$

The graph at right show several members of the family of the antiderivatives of x^2.

WHAT GRAPHICAL CONSEQUENCE DOES THE +*C* HAVE ON THE SOLUTION CURVES?

Example 1:

Find the general antiderivatives of each of the following using you knowledge of how to find derivatives.

(a) $f(x) = 2x$ (b) $f'(x) = x$ (c) $F'(x) = \frac{2}{3}x^{\frac{4}{7}}$ (d) $g'(x) = \frac{1}{x^2}$ (e) $\frac{dy}{dx} = \cos x$

Knowing how to find a derivative of different types of functions will help you find antiderivatives.

Table of Antiderivative Formulas

Function	General antiderivative		Function	General antiderivative
$cf(x)$	$cF(x)+C$		$\csc^2 x$	$-\cot x+C$
$f(x)\pm g(x)$	$F(x)\pm G(x)+C$		$\sec x\tan x$	$\sec x+C$
$x^n,\ n\neq -1$	$\frac{1}{n+1}x^{n+1}+C$		$\csc x\cot x.$	$-\csc x+C$
$\frac{1}{x}$	$\ln\lvert x\rvert+C$		$\frac{1}{\sqrt{1-x^2}}$	$\sin^{-1}x+C$
e^x	e^x+C		$\frac{1}{1+x^2}$	$\tan^{-1}x+C$
$\cos x$	$\sin x+C$		$\frac{1}{x\sqrt{x^2-1}}$	$\sec^{-1}\lvert x\rvert+C$
$\sin x$	$-\cos x+C$			
$\sec^2 x$	$\tan x+C$			

Example 2:

Find all functions g such that $g'(x)=4\sin x+\frac{2x^4-\sqrt{x}+x}{x}-\frac{7x\csc^2 x+1}{x}$.

Definition

A **differential equation** is an equation that has a derivative in it. **Solving a differential equation** involves finding the original function from which the derivative came. The **general solution** involves $+C$. The **particular solution** uses an initial condition to find the specific value of C.

Example 3:
Solve the differential equation $f'(x)=3x^2+1$ if $f(2)=-3$. Find both the general and particular solutions.

Example 4:
Find the particular solution to the following differential equation if $\frac{dy}{dx}=e^x+20\left(1+x^2\right)^{-1}$ and $y(0)=-2$.

Example 5:
Find the particular solution to the following differential equation if $\frac{d^2y}{dx^2}=12x^2+6x-4$ and

(a) $y'(1)=3$ and $y(0)=-6$

(b) $y(0)=4$ and $y(1)=1$.

When we are asked to take the derivative of an expression, we have the verb notation

$$\frac{d}{dx}\left[f(x)\right]=$$

We now need an equivalent verb expression that indicates that we find the antiderivative. It is called the indefinite integral.

Here's the anatomy of an indefinite integral:

Integral symbol

$$\int f(x)dx$$

Variable of integration

Integrand

*Because the indefinite integral gives the antiderivative, integration and antidifferentiation are mathematical synonyms, and an indefinite integral is equivalent to a general antiderivative.

Sometimes we need to manipulate our integrand into something more recognizable.

Example 6:
Evaluate each of the following:

(a) $\int\left[\frac{5\sqrt{1-x^2}}{3-3x^2}\right]dx$ (b) $\int\frac{\sin t}{\cos^2 t}dt$ (c) $\int\left(\tan^2 p+4\right)dp$

(d) $\int 3\cos^2\left(\frac{m}{2}\right)dm$ (e) $\int z^3\left(3-2z\right)^2 dz$ (f) $\int\left[\frac{x^2-5x-14}{x-7}\right]dx$